1. Black body Radiation

A black body is one that absorbs all the EM radiation (light...) that strikes it. To stay in thermal equilibrium, it must emit radiation at the same rate as it absorbs it so a black body also radiates well.

By considering plates in thermal equilibrium it can be shown that the emissive power over the absorption coefficient must be the same as a function of wavelength, even for plates of different materials.

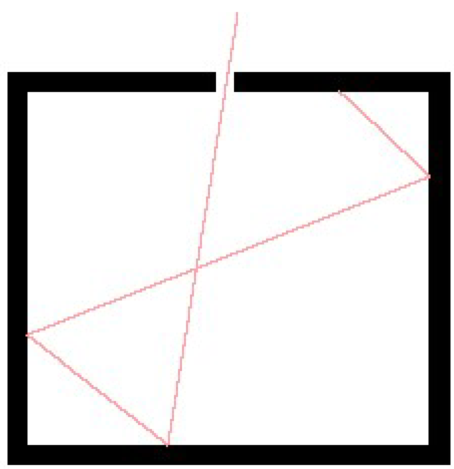
$$\frac{E_1(\lambda,T)}{A_1(\lambda)}=\frac{E_2(\lambda,T)}{A_2(\lambda)}$$

It there were differences, there could be a net energy flow from one plate to the other, violating the equilibrium condition.

A black body is one that absorbs all radiation incident upon it.

$A_{BB} = 1$

Thus, the black body Emissive power, E(ν, T), is universal and can be derived from first principles. A good example of a black body is a cavity with a small hole in it. Any light incident upon the hole goes into the cavity and is essentially never reflected out since it would have to undergo a very large number of reflections off walls of the cavity. If we make the walls absorptive (perhaps by painting them black), the cavity makes a perfect black body.



There is a simple relation between the energy density in a cavity, u(ν, T), and the black body emissive power of a black body which simply comes from an analysis of how much radiation, traveling at the speed of light, will flow out of a hole in the cavity in one second.

E(ν, T) = c/4 u(ν, T)

Rayleigh and Jeans calculated t he energy density (in EM waves) inside a cavity and hence the emission spectrum of a black body. Their calculation was based on simple EM theory and equipartition. It not only did not agree with data; it said that all energy would be instantly radiated away in high frequency EM radiation. This was called the ultraviolet catastrophe.

$$u(\nu,T)=\frac{8\pi\nu^2}{c^3}kT$$

Plank found a formula that fit the data well at both long and short wavelength.

$$u(\nu,T)=\frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT}-1}$$

His formula fit the data so well that he tried to find a way to derive it. In a few months he was able to do this, by postulating that energy was emitted in quanta with E = hν. Even though there are a very large number of cavity modes at high frequency, the probability to emit such high energy quanta vanishes exponentially according to the Boltzmann distribution. Plank thus suppressed high frequency radiation in the calculation and brought it into agreement with experiment. Note that Plank's Black Body formula is the same in the limit that hν << kT but goes to zero at large ν while the Rayleigh formula goes to infinity.

So the emissive power per unit area is

$$u(\nu,T)=\frac{2\pi\nu^2}{c^2}\frac{h\nu}{e^{h\nu/kT}-1}$$

We can integrate this over frequency to get the total power radiated per unit area.

$R(T)=\frac{c\pi^2}{60(hc)^3}\frac{h\nu}{e^{h\nu/kT}-1}$ = (5.67 × 10^{-8} W/m^2/ °K^4) T^4

Stefan Boltzmann Law:

A number of attempts aimed at explaining the origin of the continuous character of this radiation were carried out. The most serious among such attempts, and which made use of classical physics, were due to Wilhelm Wien in 1889 and Rayleigh in 1900. In 1879 J. Stefan found *experimentally* that the total intensity (or the total power per unit surface area) radiated by a glowing object of temperature T is given by

$P = a\sigma T^4$

which is known as the Stefan–Boltzmann law, where 5.67×10^{-8} W/m^2/ °K^4 is the Stefan–Boltzmann constant, and a is a coefficient which is less than or equal to 1; in the case of a blackbody $a = 1$. Then in 1884 Boltzmann provided a *theoretical* derivation for Stefan's experimental law by combining thermodynamics and Maxwell's theory of electromagnetism.

2. Wien's energy density distribution

Using thermodynamic arguments, Wien took the Stefan–Boltzmann law and in 1894 he extended it to obtain the energy density per unit frequency of the emitted blackbody radiation:

$$u(\nu, T) = A\nu^3 e^{-\beta\nu/T}$$

where A and β are empirically defined parameters (they can be adjusted to fit the experimental data). **Note:** $u(\nu, T)$ has the dimensions of an energy per unit volume per unit frequency; its SI units are Jm^{-3} Hz^{-1}. Although Wien's formula fits the high-frequency data remarkably well, it fails badly at low frequencies.

3. Rayleigh's energy density distribution

In his 1900 attempt, Rayleigh focused on understanding the nature of the electromagnetic radiation inside the cavity. He considered the radiation to consist of standing waves having a temperature T with nodes at the metallic surfaces. These standing waves, he argued, are equivalent to harmonic oscillators, for they result from the harmonic oscillations of a large number of electrical charges, electrons,that are present in the walls of the cavity. When the cavity is in thermal equilibrium, the electromagnetic energy density inside the cavity is equal to the energy density of the charged particles in the walls of the cavity; the average total energy of the radiation leaving the cavity can be obtained by multiplying the average energy of the oscillators by the number of modes (standing waves) of the radiation in the frequency interval ν to $\nu+d\nu$:

$$N(\nu) = \frac{8\pi\nu^2}{c^3}$$

where $c = 3 \times 10^8$ ms^{-1} is the speed of light; the quantity $(8\pi\, \nu^2/c^3)d\nu$ gives the number of modes of oscillation per unit volume in the frequency range ν to $\nu+d\nu$. So the electromagnetic energy density in the frequency range ν to $\nu+ d\nu$ is given by

$$u(\nu, T) = N(\nu) < E > = \frac{8\pi\nu^2}{c^3} < E >$$

where $< E >$ is the average energy of the oscillators present on the walls of the cavity (or of the electromagnetic radiation in that frequency interval); the temperature dependence of $u(\nu, T)$ is buried in $< E >$.

4. Planck's energy density distribution

By devising an ingenious scheme—interpolation between Wien's rule and the Rayleigh–Jeans rule—Planck succeeded in 1900 in avoiding the ultraviolet catastrophe and proposed an accurate description of blackbody radiation. In sharp contrast to Rayleigh's assumption that a standing wave can exchange *any* amount (continuum) of energy with matter, Planck considered that the energy exchange between radiation and matter must be *discrete*. He then *postulated* that the energy of the radiation (of frequency ν) emitted by the oscillating charges (from the walls of the cavity) must come *only* in *integer multiples* of $h\nu$:

$E = nh\nu \qquad n = 0, 1, 2, 3\ldots\ldots$

where h is a universal constant and $h\nu$ is the energy of a *"quantum"* of radiation (ν represents the frequency of the oscillating charge in the cavity's walls as well as the frequency of the radiation emitted from the walls, because the frequency of the radiation emitted by an oscillating charged particle is equal to the frequency of oscillation of the particle itself). That is, the energy of an oscillator of natural frequency ν (which corresponds to the energy of a charge oscillating with a frequency ν) must be an *integral multiple* of $h\nu$; note that $h\nu$ is not the same for all oscillators, because it depends on the frequency of each oscillator. Classical mechanics, however, puts no restrictions whatsoever on the frequency, and hence on the energy, an oscillator can have. The energy of oscillators, such as pendulums, mass–spring systems, and electric oscillators, varies continuously in terms of the frequency. Equation is known as *Planck's quantization rule* for energy or *Planck's postulate*.

So, assuming that the energy of an oscillator is quantized, Planck showed that the *correct* thermodynamic relation for the average energy can be obtained by merely replacing the integration—that corresponds to an energy continuum—by a *discrete* summation corresponding to the discreteness of the oscillators' energies:

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} nh\nu e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-nh\nu/kT}} = \frac{h\nu}{e^{h\nu/kT} - 1},$$

and the energy density per unit frequency of the radiation emitted from the hole of a cavity is given by

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT} - 1}$$

This is known as *Planck's distribution*.

We can rewrite Planck's energy density to obtain the energy density per unit wavelength.

$$\boxed{\tilde{u}(\lambda, T) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}.}$$